

# Inflationary universe in loop quantum cosmology

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**ABSTRACT:** Loop quantum cosmology provides a nice solution of avoiding the big bang singularity through a big bounce mechanism in the high energy region. In loop quantum cosmology an inflationary universe is emergent after the big bounce, no matter what matter component is filled in the universe. A super-inflation phase without phantom matter will appear in a certain way in the initial stage after the bounce; then the universe will undergo a normal inflation stage. We discuss the condition of inflation in detail in this framework. Also, for slow-roll inflation, we expect the imprint from the effects of the loop quantum cosmology should be left in the primordial perturbation power spectrum. However, we show that this imprint is too weak to be observed.

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## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. Effective dynamics in loop quantum cosmology and big bounce of the universe</b>	<b>3</b>
<b>3. Inflationary universe after big bounce</b>	<b>5</b>
3.1 Super-inflation and effective quintom	5
3.2 Condition for inflation	8
3.3 $e$ -foldings	10
3.4 Slow-roll inflation and primordial perturbations	11
3.5 Can LQC effects be observable?	14
<b>4. Conclusions</b>	<b>15</b>

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## 1. Introduction

The inflation paradigm provides an exquisite explanation for some severe problems of the cosmological standard model by positing an epoch of accelerated expansion in the early universe [1, 2]. This accelerated period of expansion also generates superhorizon fluctuations and thus predicts an almost scale-invariant density perturbation power spectrum, which has received strong observational support from the measurement of the temperature fluctuation in the cosmic microwave background (CMB) radiation [3, 4, 5]. Conceptually, however, the inflationary scenario is incomplete due to the existence of the big bang singularity [6]. Einstein’s classical theory of general relativity (GR) breaks down near such a singularity since quantum effects are expected to be important at very high energies in the early universe. So, the classical theory of GR has to be replaced by some theoretical framework of quantum gravity which should remain well defined even at very high curvatures.

Loop quantum gravity (LQG) is a leading nonperturbative background independent approach to quantizing gravity [7]. The underlying geometry in LQG is discrete and the continuum spacetime can be obtained from the quantum geometry in a large eigenvalue limit. Loop quantum cosmology (LQC) focuses on symmetry reduced models (with homogeneous and isotropic space) but inherits quantization scheme and techniques from LQG [8]. Investigations of LQC has led to important insights on the resolution of singularities in various situations [9, 10, 11]. Within the framework of LQC, some long-standing issues concerning the quantum nature of the big-bang are resolved in the context of homogeneous

and isotropic universe with a scalar field. Using extensive analytical and numerical methods, the analysis of the evolution of the semiclassical states for a spatially flat universe has shown that the universe has a pre-big-bang branch, joined deterministically to the post-big-bang branch by a quantum bounce in the deep Planck regime through the LQC evolution [12, 13, 14]. Thanks to the nonperturbative background independent methods of LQC, the idea of the nonsingular bounce can be realized in a natural fashion.

An important feature of LQC is that the underlying dynamics is governed by a discrete quantum difference equation of quantum geometry. An effective description of quantum dynamics, however, can be obtained by applying geometric methods to quantum mechanics, where the Hilbert space is treated as an infinite dimensional phase space which has a structure of a fiber bundle. Using semiclassical states one can construct an effective Hamiltonian description on a continuum spacetime which incorporates the leading quantum corrections to the classical dynamics and has been shown to be a very good approximation to the quantum dynamics [13, 14, 15]. Intriguingly, one can obtain an modified Friedmann equation from the effective Hamiltonian constraint, which can be used to investigate the role of nonperturbative quantum correction conveniently. It is remarkable that the quantum geometric effects lead to a  $\rho^2$  modification to the Friedmann equation at the scales when  $\rho$  becomes comparable to a critical density  $\rho_{\text{crit}}$  which is close to the Planck density ( $\rho_{\text{crit}} \approx 0.82\rho_{\text{Pl}}$ ) [13, 14, 16, 17, 18]. The modified term in the Friedmann equation is negative definite, which implies a bounce when the energy density hits the critical value; this feature resolves the classical big-bang singularity problem and is in accordance with the result from the quantum evolution in LQC.

Inflation begins after the big bounce when the quantum geometric effects are dominant. Some questions naturally arise: What influence does the quantum geometry make on the inflation? Can any matter (with arbitrary equation of state) lead to inflation under the consideration of LQC? What conditions are needed for inflation in the effective LQC? Can the LQC effects provide sufficient number of  $e$ -foldings for inflation? Since inflation happens at the very early time, the effects of quantum gravity should in principle leave an imprint on the primordial spectrum of perturbations, because the wavelengths of perturbations emerged from short distances in early stages of inflation are stretched to the cosmic scales observable today by the rapid expansion during inflation. So, relevant questions are: How the LQC effects affect the primordial power spectra? Can LQC leave imprint on the CMB sky? Can we observe it?

This work seeks to answer these questions using the effective theory of LQC. This paper is organized as follows. In the next section we briefly review the effective theory of LQC and describe the quantum big-bounce of the universe generated by loop quantum dynamics. In section 3, we discuss some issues of inflation using the modified Friedmann equation in the effective theory of LQC. We analyze the condition of inflation in detail taking the LQC effects into account; we discuss the  $e$ -folding number relevant to the quantum gravity effects; we calculate the primordial power spectra of density perturbations and gravitational waves, spectral indices and tensor-to-scalar ratio in the slow-roll inflation when considering the influences of the LQC; and we also analyze the observational constraints on the LQC imprint on the CMB sky. We give conclusions in section 4.

## 2. Effective dynamics in loop quantum cosmology and big bounce of the universe

LQG is a canonical quantization of gravity based upon Ashtekar-Barbero connection variables. The phase space of classical GR in LQG is spanned by  $SU(2)$  connection  $A_a^i$  and the triad  $E_i^a$  on a 3-manifold  $\mathcal{M}$  (labels  $a$  and  $i$  denote space and internal indices respectively), which are two conjugate variables encoding curvature and spatial geometry, respectively. Likewise, LQC is a canonical quantization of homogeneous spacetimes based upon techniques used in LQG. In LQC, due to the symmetries of the homogeneous and isotropic spacetime, the phase space structure is simplified, i.e., the connection is determined by a single quantity labeled  $\mathfrak{c}$  and likewise the triad is determined by a parameter  $\mathfrak{p}$ . The variables  $\mathfrak{c}$  and  $\mathfrak{p}$  are canonically conjugate with Poisson bracket  $\{\mathfrak{c}, \mathfrak{p}\} = \gamma\kappa/3$ , where  $\kappa = 8\pi G$  ( $G$  is the Newton's gravitational constant) and  $\gamma$  is the dimensionless Barbero-Immirzi parameter which is set to be  $\gamma \approx 0.2375$  by the black hole thermodynamics in LQG [19]. For the spatially flat model of cosmology, the new variables have the relations with the metric components of the Friedmann-Robertson-Walker (FRW) universe as

$$\mathfrak{c} = \gamma\dot{a}, \quad \mathfrak{p} = a^2, \quad (2.1)$$

where  $a$  is the scale factor of the universe. Classically in terms of the connection-triads variables the Hamiltonian constraint is given by

$$\mathcal{H}_{\text{cl}} = -\frac{3\sqrt{\mathfrak{p}}}{\kappa\gamma^2}\mathfrak{c}^2 + \mathcal{H}_{\text{M}}, \quad (2.2)$$

where  $\mathcal{H}_{\text{M}}$  is the matter Hamiltonian.

The elementary variables used for quantization in LQC are the triads and holonomies of the connection. The holonomy over an edge of a loop is defined as  $h_i(\mu) = \cos(\mu\mathfrak{c}/2) + 2\sin(\mu\mathfrak{c}/2)\tau_i$ , where  $\tau_i$  is related to Pauli spin matrices as  $\tau_i = -i\sigma_i/2$  and dimensionless  $\mu$  is related to the physical length of the edge over which holonomy is evaluated (note that  $\mu$  is also the eigenvalue of the triad operator  $\hat{\mathfrak{p}}$ ). In the Hamiltonian formulation for homogeneous and isotropic spacetime, the dynamical equations can be determined completely by the Hamiltonian constraint. Under quantization, the Hamiltonian constraint gets promoted to an operator and the quantum wave functions are annihilated by the operator of the Hamiltonian constraint. In LQC, it is expected that modifications due to LQC effects will appear in the Hamiltonian constraint, and from the modified Hamiltonian constraint the effective Friedmann constraint will be derived. In quantization the Hamiltonian constraint operator is obtained by promoting the holonomies and the triads to the corresponding operators. Consequently, this leads to a discrete quantum difference equation, which indicates that the underlying geometry in LQC is discrete [9, 10]. Interestingly, the solutions of this difference equation are nonsingular.

So far we see that the underlying dynamics in LQC is governed by a discrete quantum difference equation in quantum geometry. However, an effective Hamiltonian description on a continuum spacetime can be constructed by using semiclassical states, which has been shown to very well approximate the quantum dynamics [13, 14]. This analysis reveals that

on backward evolution of our expanding phase of the universe, the universe bounces at a critical density (near the big bang singularity) into a contracting branch [12, 18]. Thus the classical singular problem can be successfully overcome within the context of LQC by a non-singular bounce. In addition, the effective equations for the modified Friedmann dynamics can be derived from the effective Hamiltonian constraint with loop quantum modifications, which can be used to investigate the role of nonperturbative quantum corrections. An important feature for the modified dynamics is that a  $\rho^2$  term which is relevant in the high energy regime is included in the classical Friedmann equation. The modified term is negative definite implying a bounce when the energy density reaches a critical value on the order of the Planck density.

The effective Hamiltonian constraint, to leading order, is given by [15]

$$\mathcal{H}_{\text{eff}} = -\frac{3}{\kappa\gamma^2\bar{\mu}^2}a\sin^2(\bar{\mu}\mathfrak{c}) + \mathcal{H}_{\text{M}}, \quad (2.3)$$

where  $\bar{\mu}$  is the kinematical length of the edge of a square loop which has the area given by the minimum eigenvalue of the area operator in LQG; the area is  $\mathcal{A} = \bar{\mu}^2 a^2 = \alpha l_{\text{Pl}}^2$ , where  $\alpha$  is of the order unity and  $l_{\text{Pl}} = \sqrt{G}$  is the Planck length. The modified Friedmann equation can then be derived by using the Hamilton's equation for  $\mathfrak{p}$ ,

$$\dot{\mathfrak{p}} = \{\mathfrak{p}, \mathcal{H}_{\text{eff}}\} = -\frac{\kappa\gamma}{3}\frac{\partial\mathcal{H}_{\text{eff}}}{\partial\mathfrak{c}} = \frac{2a}{\gamma\bar{\mu}}\sin(\bar{\mu}\mathfrak{c})\cos(\bar{\mu}\mathfrak{c}), \quad (2.4)$$

which combined with Eq. (2.1) yields the rate of change of the scale factor

$$\dot{a} = \frac{1}{\gamma\bar{\mu}}\sin(\bar{\mu}\mathfrak{c})\cos(\bar{\mu}\mathfrak{c}). \quad (2.5)$$

Furthermore, the vanishing of the Hamiltonian constraint,  $\mathcal{H}_{\text{eff}} \approx 0$ , implies

$$\sin^2(\bar{\mu}\mathfrak{c}) = \frac{\kappa\gamma^2\bar{\mu}^2}{3a}\mathcal{H}_{\text{M}}. \quad (2.6)$$

Combining Eqs. (2.5) and (2.6) yields the effective Friedmann equation for the Hubble rate  $H = \dot{a}/a$ ,

$$H^2 = \frac{\kappa}{3}\rho\left(1 - \frac{\rho}{\rho_{\text{crit}}}\right), \quad (2.7)$$

with the critical density given by

$$\rho_{\text{crit}} = \frac{\sqrt{3}}{16\pi^2\gamma^3}\rho_{\text{Pl}}, \quad (2.8)$$

where  $\rho_{\text{Pl}} = G^{-2}$  is the Planck density. The modified Friedmann equation provides an effective description for LQC which very well approximates the underlying discrete quantum dynamics. The nonperturbative quantum geometric effects are manifested in the modified Friedmann equation with a  $\rho^2$  correction term. The negative definition of the  $\rho^2$  term implies that the Hubble parameter vanishes when  $\rho = \rho_{\text{crit}}$  and the universe experiences a turnaround in the scale factor. When  $\rho \ll \rho_{\text{crit}}$ , the modifications to the Friedmann

equation become negligible, and the standard Friedmann equation is recovered. It is remarkable that the origin of  $\rho_{\text{crit}}$  is purely quantum, since in the classical limit  $\hbar \rightarrow 0$  one has  $\rho_{\text{crit}} \rightarrow \infty$ . In addition, it should be noted that, interestingly,  $\rho^2$  modifications also appear in string inspired braneworld scenarios and it has been shown that there exist interesting dualities between the two frameworks [16]. Such modifications in braneworlds, however, are usually positive definite so that a bounce is absent, unless the existence of two timelike extra dimensions is assumed [20, 21].

The modified Friedmann equation (2.7) along with the conservation law

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (2.9)$$

provides a pre-big-bang picture with a big bounce occurring when  $\rho = \rho_{\text{crit}}$ . The condition for the existence of a bounce is that the matter in the universe does not violate the null energy condition [18]. Let us consider the simplest case of matter with constant equation of state  $w$  (the definition of  $w$  is  $w = p/\rho$ ), then from Eq. (2.9) we have  $\rho \propto a^{-3(1+w)}$ . Thus the conclusion can be drawn that for  $w < -1$  a recollapse can occur and for  $w > -1$  a bounce can occur. This can be easily understood. Matter with equation of state  $w < -1$  violates the null energy condition and hence has increasing energy density as the universe expands. Thus when the energy density reaches the quantum critical value a recollapse appears and the universe begins contracting [22]. On the other hand, matter with equation of state  $w > -1$  has increasing energy density as the universe contracts and therefore a quantum bounce will occur when the quantum critical density is hit. So it is clear that generically a phantom field can not have a quantum bounce in the early universe [18]. In this paper we will only consider the “normal” matter (or canonical scalar field) without null energy condition violation, and we will focuss on the post-big-bounce stage of the universe.

### 3. Inflationary universe after big bounce

In this section we shall investigate inflation within the framework of the effective theory of LQC. In LQC, a quantum bounce plays a role of junction of pre-big-bang branch and post-big-bang branch, so the classical big-bang singularity is erased by the quantum big-bounce. Naturally, the quantum bounce becomes the initial condition of the subsequent inflation. It is evident that the quantum gravity effects will play a significant role in the “primary inflation”, however, what is important is that whether the quantum geometry effects can leave influences on the “observable inflation” with the last 60  $e$ -foldings. We shall discuss some topics of interest of inflation in what follows.

#### 3.1 Super-inflation and effective quintom

Though the issue of super-inflation in LQC has been discussed widely in the literature (see, e.g., [16, 18, 23, 24]), we still make some analyses here for maintaining this paper self-contained. Besides, we discuss the effective behavior of a quintom matter in the inflation process in this subsection.

According to the effective description of the quantum dynamics, the Friedmann equation is modified to include a  $\rho^2$  term relevant to high energy region, shown as Eq. (2.7).

Thus, using the Friedmann equation (2.7) and the conservation law (2.9), we have

$$\dot{H} = -\frac{1}{2}(\rho + p) \left(1 - \frac{2\rho}{p}\right), \quad (3.1)$$

where  $p$  is the pressure of the matter filled in the universe, and we have set  $\kappa = 1$  for convenience (this convention will be used in the most situations hereafter). It is obvious that  $\dot{H}$  will be always larger than zero when the energy density is in the range of  $\rho_{\text{crit}}/2 < \rho < \rho_{\text{crit}}$ , provided that  $\rho + p > 0$ . This implies that a super-inflation phase occurs after the bounce when the energy is very high. Since  $\rho + p > 0$  (i.e.  $w > -1$ ), there is no phantom matter in the universe; so the super-inflation is purely due to the quantum geometry effects in LQC.<sup>1</sup> As shown in the previous section, matter with  $w < -1$  will not lead to big-bounce, thus in this paper we only consider the matter with  $w > -1$  which does not violate the weak energy condition. From Eqs. (2.7) and (3.1), we derive

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6} \left\{ \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}}\right) + 3 \left[ p \left(1 - \frac{2\rho}{\rho_{\text{crit}}}\right) - \frac{\rho^2}{\rho_{\text{crit}}} \right] \right\}. \quad (3.2)$$

Comparing to the classical form of the equation, it is convenient to define the effective energy density and pressure

$$\rho_{\text{eff}} = \rho \left(1 - \frac{\rho}{\rho_{\text{crit}}}\right), \quad (3.3)$$

$$p_{\text{eff}} = p \left(1 - \frac{2\rho}{\rho_{\text{crit}}}\right) - \frac{\rho^2}{\rho_{\text{crit}}}, \quad (3.4)$$

then Eq. (3.2) can be written as

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho_{\text{eff}} + 3p_{\text{eff}}). \quad (3.5)$$

Also, we have

$$H^2 = \frac{1}{3}\rho_{\text{eff}}, \quad (3.6)$$

$$\dot{H} = -\frac{1}{2}(\rho_{\text{eff}} + p_{\text{eff}}). \quad (3.7)$$

Given the effective energy density and pressure, the effective equation of state is defined naturally as

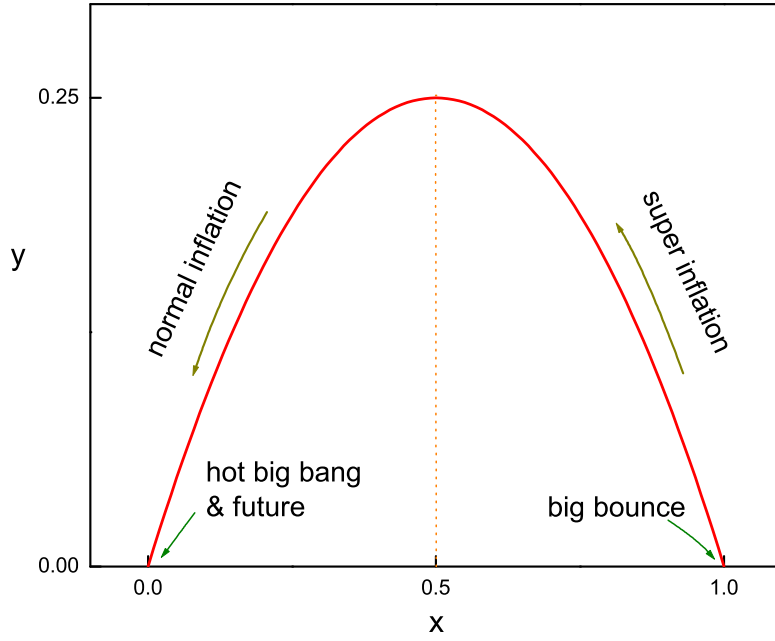
$$w_{\text{eff}} = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = \frac{w(1 - 2x) - x}{1 - x}, \quad (3.8)$$

where  $x$  is defined as dimensionless density,  $x = \rho/\rho_{\text{crit}}$ , so we have  $0 < x < 1$ .

From Eq. (3.8), we see that provided that the dimensionless density  $x$  is placed in the range  $1/2 < x < 1$ , the effective equation of state will be smaller than  $-1$ , namely  $w_{\text{eff}} < -1$ , giving rise to the phase of super-inflation. The super-inflation will last to

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<sup>1</sup>It should be noted that super-inflation occurs after any bounce by definition. Bounce is a point with  $H = 0$  ( $\dot{a} = 0$ ),  $\ddot{a} > 0$ , so if we have a bounce, we should have some period of super-inflation (i.e., growth of  $H$  during expansion) after, independently of a particular theory which causes this bounce. The bounce discussed in this paper originates from the effects of LQC, so does the super-inflation.

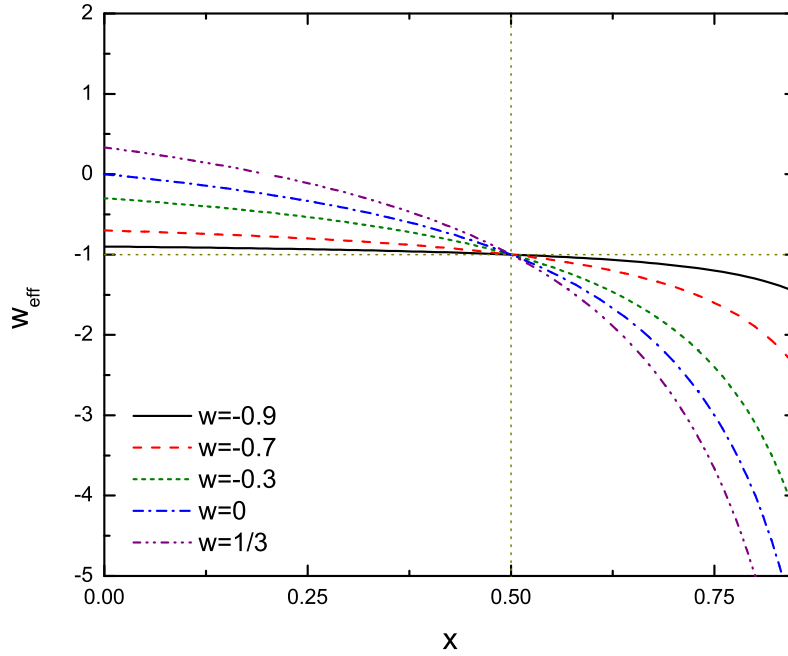


**Figure 1:** Illustration of inflationary universe in the effective theory of LQC. Here  $y$  denotes  $\rho_{\text{eff}}/\rho_{\text{crit}}$  and  $x$  denotes  $\rho/\rho_{\text{crit}}$ . The plot shows that after the big-bounce a super-inflation begins due to the quantum gravity effects during  $1/2 < x < 1$ , and subsequently the universe undergoes a normal-inflation until some reheating process initiates the “hot-big-bang”. The dimensionless effective energy density  $y$  has a maximum  $y_{\text{max}} = 1/4$  at the end of super-inflation. The value of  $y$  first increases and then decreases, implying an effective behavior of “quintom”, which is totally due to the influence of quantum geometry effects.

$w_{\text{eff}} = -1$  when  $x = 1/2$  and the effective energy density of the universe  $\rho_{\text{eff}}$  achieves its maximum value,  $\rho_{\text{crit}}/4$ . The inflationary process is sketched in Fig. 1 which plots the rewritten Eq. (3.3),  $y = x(1-x)$ , where  $y = \rho_{\text{eff}}/\rho_{\text{crit}}$  and  $x = \rho/\rho_{\text{crit}}$ . This figure explicitly shows that after the quantum big-bounce of the universe a super-inflation begins due to the quantum gravity effects during  $1/2 < x < 1$ , and subsequently the universe undergoes a normal-inflation stage until some reheating process initiates the “hot-big-bang” epoch. It is worthwhile to note that the difference between the conceptions of “classical big-bang singularity” and “hot-big-bang epoch” should be distinct. The classical big-bang singularity has been replaced by a quantum bounce in the theory of LQC, and the hot-big-bang epoch is originated from some reheating mechanism of the theory of inflation. The hot-big-bang epoch and future evolution era are expected to happen at the low energy region (i.e.,  $x \ll 1$ ) where the quantum geometry effects can be negligible. From Fig. 1 it can be seen that the value of  $y$  first increases and then decreases, which implies that the effective behavior of the universe under the quantum gravity domination resembles a “quintom” (for quintom see e.g. [25]). The key feature of quintom is that its equation of state can evolve across the cosmological-constant boundary (or “phantom divide”)  $w = -1$ . The effective equation of state of the universe in LQC, Eq. (3.8), just possesses this significant characteristic, see



Fig. 2. In the examples of Fig. 2, we endow values to the equation of state of matter in the universe,  $w$ , as  $-0.9$ ,  $-0.7$ ,  $-0.3$ ,  $0$  and  $1/3$ , respectively. It is shown in this figure that  $w_{\text{eff}}$  will evolve across the phantom divide  $w = -1$  no matter what matter component is filled in the universe. Even though the dominant matter component is dust-like matter ( $w = 0$ ) or radiation-like matter ( $w = 1/3$ ), the super-inflation and the subsequent normal inflation will both happen deterministically because of the effective quintom behavior of the universe due to the quantum gravity nature of the big-bounce. It is remarkable that recently in Ref. [26] the authors pointed out that a bouncing universe should be filled with quintom matter. Intriguingly, we show here that an effective quintom behavior is emergent in the bouncing universe purely due to the effects of quantum geometry in LQC.



**Figure 2:** The effective equation-of-state  $w_{\text{eff}}$  versus the dimensionless density parameter  $x$ . The equation-of-state of matter in the universe,  $w$ , is assumed to be constant for simplicity, and its value is taken to be  $-0.9$ ,  $-0.7$ ,  $-0.3$ ,  $0$  and  $1/3$ , for example. It is shown that  $w_{\text{eff}}$  evolves across the “phantom divide”  $w = -1$ , which is the significant characteristic of the “quintom” matter. Concretely,  $w_{\text{eff}} < -1$  when  $1/2 < x < 1$ , and  $w_{\text{eff}} > -1$  when  $0 < x < 1/2$ .

### 3.2 Condition for inflation

In this subsection we study the condition of inflation in detail within the framework of LQC. In the previous subsection we see that after the big-bounce a super-inflation is emergent naturally and a normal inflation occurs subsequently. The emergence of the super-inflation is purely a phenomena of quantum gravity, which is irrelevant to what matter is filled in the universe. The subsequent normal inflation will also happen deterministically, but whether it can last sufficient  $e$ -foldings is relevant to the matter component.

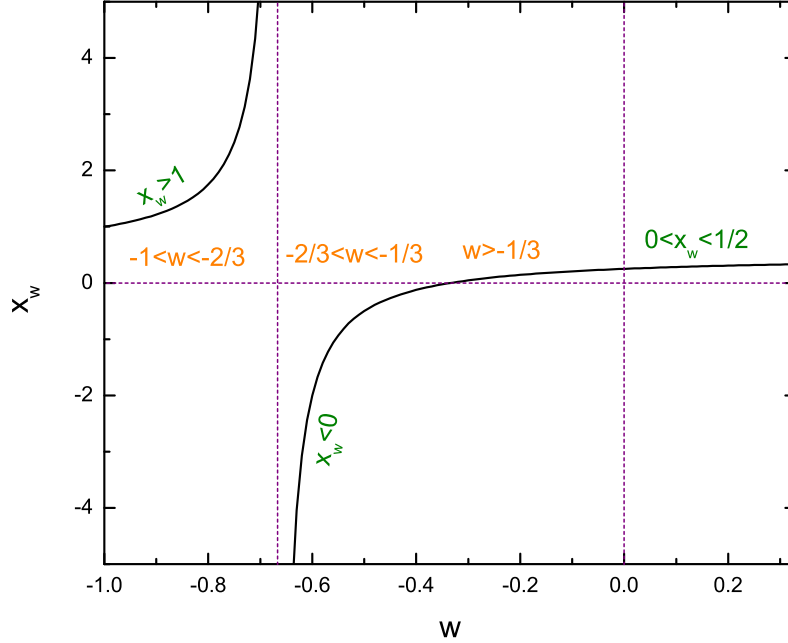
Generically, the criteria of judgement for inflation is attributed to  $\epsilon_H < 1$ , where the parameter  $\epsilon_H$  is defined as  $\epsilon_H = -\dot{H}/H^2$ . Using Eqs. (2.7) and (3.1), we can easily derive

$$\epsilon_H = \frac{3}{2}(1+w)\frac{1-2x}{1-x}. \quad (3.9)$$

Evidently, the condition  $\epsilon_H < 1$  is totally equivalent to the condition  $w_{\text{eff}} < -1/3$ . Solving the inequality  $\epsilon_H < 1$  or  $w_{\text{eff}} < -1/3$  yields a set of two inequalities:  $x > x_w$  for  $w > -2/3$ ,  $x < x_w$  for  $w < -2/3$ , where  $x_w$  is defined as

$$x_w = \frac{1+3w}{4+6w}. \quad (3.10)$$

However, since  $0 < x < 1$ , we have to analyze whether or not  $x_w$  is located in the range  $x \in (0, 1)$ . We find from Eq. (3.10) that  $0 < x_w < 1/2$  for  $w > -1/3$ ,  $x_w < 0$  for  $-2/3 < w < -1/3$ , and  $x_w > 1$  for  $-1 < w < -2/3$ . For sketching the cases of  $x_w$  values clearly, we plot Eq. (3.10) in Fig. 3.



**Figure 3:** The plot of  $x_w$  versus  $w$ . It is shown that  $0 < x_w < 1/2$  for  $w > -1/3$ ,  $x_w < 0$  for  $-2/3 < w < -1/3$ , and  $x_w > 1$  for  $-1 < w < -2/3$ .

Given  $0 < x < 1$  as well as the analysis of  $x_w$ , the condition of  $x > x_w$  for  $w > -2/3$  is divided into two parts:  $x > x_w$  for  $w > -1/3$ , and  $x > 0$  for  $-2/3 < w < -1/3$ ; the condition of  $x < x_w$  for  $-1 < w < -2/3$  becomes  $x < 1$  for  $-1 < w < -2/3$ . Therefore, we have the condition for inflation as follows

$$\begin{aligned} 0 < x_w < x < 1, & \text{ for } w > -1/3; \\ 0 < x < 1, & \text{ for } -1 < w < -1/3. \end{aligned} \quad (3.11)$$

It is obvious that when  $-1 < w < -1/3$  inflation will always happen which is in accordance with the usual cases of inflation. However, when  $w > -1/3$ , inflation will also take place for period of time, which is different from the usual inflation cases and is thus purely due to the influence of the quantum geometry. In particular, the inflation process is divided into two stages, super-inflation and normal-inflation. For super-inflation, the condition is

$$1/2 < x < 1, \text{ for any } w > -1, \quad (3.12)$$

which has been discussed in the previous subsection. For normal inflation, the condition is

$$\begin{aligned} 0 < x_w < x < 1/2, \text{ for } w > -1/3; \\ 0 < x < 1/2, \text{ for } -1 < w < -1/3. \end{aligned} \quad (3.13)$$

So far we see that the quantum geometry effects lead to intriguing phenomena in the inflationary stage after the big-bounce in LQC. First, any matter with  $w > -1$  can give rise to super-inflation after the bounce when  $1/2 < x < 1$ . In addition, matter with  $w > -1/3$  can proceed to drive the subsequent normal inflation for period of time. So, even though the universe is filled in radiation-like or dust-like component at that time, inflation will still happen. These peculiar phenomena all root in the effects of quantum geometry in LQC. A question is now emergent: How many  $e$ -foldings can this LQC induced inflation last?

### 3.3 $e$ -foldings

We now discuss the issue of  $e$ -folding number in this subsection. It is of interest to know how many  $e$ -foldings the inflation can last when  $w > -1/3$  in the framework of the effective dynamics of LQC. Also, for  $-1 < w < -1/3$ , it is necessary to evaluate when the inflation should cease given the number of  $e$ -foldings.

For simplicity we assume that the equation of state of the matter in the universe is a constant. Such an assumption makes Eq. (2.9) have a simple solution,

$$\rho = \rho_{\text{crit}} \left( \frac{a}{a_{\text{crit}}} \right)^{-3(1+w)}, \quad (3.14)$$

where  $a_{\text{crit}}$  denotes the scale factor at bouncing point when the energy density approaches the critical value, so  $a_{\text{crit}}$  is the minimum scale for the universe,  $a > a_{\text{crit}}$ . According to the definition of the parameter  $x$ ,  $x = \rho/\rho_{\text{crit}}$ , we have  $x = (a/a_{\text{crit}})^{-3(1+w)}$ , then we obtain

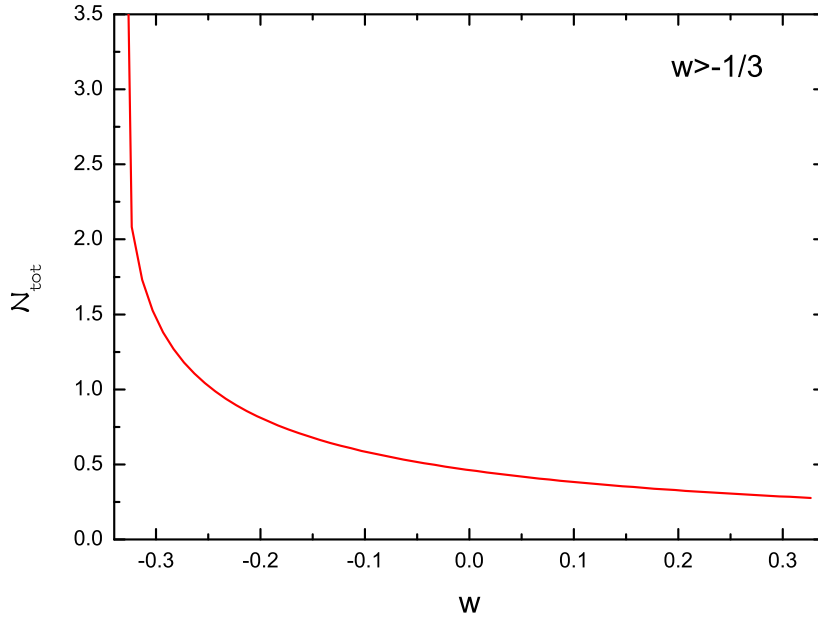
$$e^{\mathcal{N}} \equiv \frac{a}{a_{\text{crit}}} = x^{-\frac{1}{3(1+w)}}, \quad (3.15)$$

where  $\mathcal{N}$  is the number of  $e$ -foldings from the bouncing point (corresponding to  $x = 1$ ) to somewhere in inflation stage (labeled by  $x$ ).

Therefore, for cases of  $w > -1/3$ , the total number of  $e$ -foldings can be evaluated directly,

$$\mathcal{N}_{\text{tot}} = -\frac{1}{3(1+w)} \ln x_w, \quad (3.16)$$

where  $\mathcal{N}_{\text{tot}}$  represents the total  $e$ -folding number, and  $x_w$  is given by Eq. (3.10). The inflation driven by such an equation of state will terminate at  $x_w$ , so the total  $e$ -folding



**Figure 4:** The relation between the total  $e$ -folding number  $\mathcal{N}_{\text{tot}}$  and the equation-of-state  $w$ , when  $w > -1/3$ . The inflation driven by such an equation of state originates from the quantum geometry effects in LQC. The plot shows that such an inflation can last at most several  $e$ -foldings.

number is a function of  $w$ , namely  $\mathcal{N}_{\text{tot}}(w)$ , see Fig. 4. This figure shows that such an inflation can last at most several  $e$ -foldings, e.g.,  $\mathcal{N}_{\text{tot}}(1/3) = 0.275$ ,  $\mathcal{N}_{\text{tot}}(0) = 0.462$ ,  $\mathcal{N}_{\text{tot}}(-0.30) = 1.472$ , and  $\mathcal{N}_{\text{tot}}(-0.33) = 2.641$ .

For cases of  $-1 < w < -1/3$ , inflation can always proceed, thus a reheating mechanism is needed to cease the inflation and initiate the hot-big-bang era. The concrete mechanism of reheating is not a issue of concern in this paper. What is of interest for us is that one can evaluate the value of  $x$  when the inflation ends if  $w$  and  $\mathcal{N}_{\text{tot}}$  are given. The end point of inflation can be expressed as

$$x_{\text{end}} = e^{-3(1+w)\mathcal{N}_{\text{tot}}}. \quad (3.17)$$

In fact, the total number of  $e$ -folds of inflation is an unknown value. Our observational universe corresponds to roughly last 60  $e$ -folds before the end of inflation, hence at most the last 60  $e$ -folds have a directly observational effect. Inflation stage before the observational universe leaves the horizon can not be observed in principle, so this stage is called “primary inflation”. As oppose to this unobservable stage, we call the inflation stage corresponding to the observational universe the “observational inflation”. Hence, the total number of  $e$ -foldings of inflation must be larger than 60, namely  $\mathcal{N}_{\text{tot}} > 60$ , and may even reach several hundreds or more. Note that the value of  $x_{\text{end}}$  is related to the energy scale of reheating since  $x_{\text{end}} = \rho_{\text{end}}/\rho_{\text{crit}}$ .

### 3.4 Slow-roll inflation and primordial perturbations

During inflation the wavelengths of perturbations generated from short scales where quan-

tum gravity effects are important are stretched to cosmic scales by rapid expansion, thus the effects of quantum gravity should in principle leave an imprint on the primordial spectrum of perturbations. Quantum geometry effects in LQC lead to a  $\rho^2$  modification in the Friedmann equation which gives rise to a quantum bouncing solution to replace the classical big-bang singularity in the very early universe. The quantum gravity effects, however, only play a significant role in the very high energy regime, the quantum correction  $\rho^2$  term will be negligible when the energy scale is much smaller than the scale of the critical density. Still, it is expected that the imprint of quantum effects should be left on the primordial power spectrum which can be investigated through observation on the CMB sky. It should be mentioned that the stringy imprint in the primordial perturbations has been discussed in the noncommutative inflation models, see e.g., [27].

While cosmological scales are leaving the horizon, the slow-roll paradigm of inflation is practically mandatory in order to account for the near scale invariance of spectrum of the primordial curvature perturbation. We study the slow-roll inflation within the framework of the effective theory of LQC, i.e., the Friedmann equation given by Eq. (2.7). The inflation process is driven by a spatially homogeneous scalar field  $\phi$  (the inflaton) satisfying the following equation of motion,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (3.18)$$

where the prime denotes the derivative with respect to the field  $\phi$ , and the Hubble parameter  $H$  is given by Eq. (2.7). If  $\dot{\phi}^2 \ll V(\phi)$  and  $\ddot{\phi} \ll 3H\dot{\phi}$ , the scalar field will slowly roll down its potential, and the exact evolution equation (3.18) can be replaced by the slow-roll approximation,

$$\dot{\phi} = -V'/3H. \quad (3.19)$$

Under the slow-roll condition, the energy density of the scalar field approximates as  $\rho \sim V(\phi)$ , the Friedmann equation (2.7) consequently can be written as

$$H^2 = \frac{1}{3}V \left( 1 - \frac{V}{\rho_{\text{crit}}} \right). \quad (3.20)$$

For convenience one can define an effective potential for the scalar field,

$$V_{\text{eff}}(\phi) = V(\phi)(1 - \nu(\phi)), \quad (3.21)$$

where  $\nu(\phi)$  is defined as

$$\nu(\phi) = \frac{V(\phi)}{\rho_{\text{crit}}}. \quad (3.22)$$

It is obvious that  $\nu(\phi)$  describes the quantum geometry effects in LQC. The slow-roll parameters can be defined in terms of the potential and its derivatives as usual,

$$\epsilon_v = \frac{1}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_v = \frac{V''}{V}, \quad \xi_v^2 = \frac{V'V'''}{V^2}, \quad (3.23)$$

then the slow-roll condition can be expressed as  $\epsilon_v, |\eta_v| \ll 1$ . The inflation ends when the slow-roll condition ceases to be satisfied. The small change of  $e$ -foldings satisfies  $dN \equiv$

$-Hdt$ ; using Eqs. (3.19) and (3.20), one obtains the number of  $e$ -foldings of slow-roll inflation remaining at a given epoch,

$$N(\phi) = \int_{\phi_{\text{end}}}^{\phi} \frac{V_{\text{eff}}}{V'} d\phi = \int_{\phi_{\text{end}}}^{\phi} (1 - \nu) \frac{V}{V'} d\phi, \quad (3.24)$$

where  $\phi_{\text{end}}$  marks the end of slow-roll inflation.

The perturbation  $\delta\phi$  can be treated as a massless free field, and its vacuum fluctuation can be regarded as a classical quantity a few Hubble times after horizon exit. The spectrum of the field perturbation is

$$\mathcal{P}_\phi = (H/2\pi)^2. \quad (3.25)$$

The corresponding curvature perturbation is given by  $\mathcal{R} = (H/\dot{\phi})\delta\phi$ . Using Eqs. (3.19), (3.20) and (3.25), we get the amplitude of scalar perturbation as

$$\mathcal{P}_\mathcal{R} = \frac{H^2}{\dot{\phi}^2} \left( \frac{H}{2\pi} \right)^2 = \frac{V_{\text{eff}}^3}{12\pi^2 V'^2} = \frac{V^3}{12\pi^2 V'^2} (1 - \nu)^3, \quad (3.26)$$

which is evaluated at the Hubble radius crossing  $k = aH$ . Since  $H$  is slowly varying, we have  $d \ln k = d(\ln(aH)) \simeq d \ln a = Hdt$ , then from Eqs. (3.19) and (3.20), we get

$$\frac{d}{d \ln k} = -\frac{V'}{V_{\text{eff}}} \frac{d}{d\phi} = -\frac{1}{1 - \nu} \frac{V'}{V} \frac{d}{d\phi}. \quad (3.27)$$

We shall need the following expressions:

$$\frac{d\epsilon_v}{d \ln k} = \frac{1}{1 - \nu} (-2\epsilon_v \eta_v + 4\epsilon_v^2), \quad (3.28)$$

$$\frac{d\eta_v}{d \ln k} = \frac{1}{1 - \nu} (2\epsilon_v \eta_v - \xi_v^2). \quad (3.29)$$

The spectral index of the scalar perturbation and its running can thus be given,

$$n_s - 1 \equiv \frac{d \ln \mathcal{P}_\mathcal{R}}{d \ln k} = -6\epsilon_v \frac{1 - 2\nu}{(1 - \nu)^2} + 2\eta_v \frac{1}{1 - \nu}, \quad (3.30)$$

$$\alpha_s \equiv \frac{dn_s}{d \ln k} = 16\epsilon_v \eta_v \frac{1 - 2\nu}{(1 - \nu)^3} - 24\epsilon_v^2 \frac{1 - 3\nu(1 - \nu)}{(1 - \nu)^4} - 2\xi_v^2 \frac{1}{(1 - \nu)^2}. \quad (3.31)$$

Inflation also generates gravitational waves with two independent components  $h_{+, \times}$  which have the same action as a massless scalar field. Likewise, the amplitude of tensor perturbations can also be given,

$$\mathcal{P}_{\text{grav}} = 8 \left( \frac{H}{2\pi} \right)^2 = \frac{2V_{\text{eff}}}{3\pi^2} = \frac{2V}{3\pi^2} (1 - \nu), \quad (3.32)$$

which is also evaluated at the horizon exit  $k = aH$ . Then we obtain the spectral index of tensor perturbation and the scalar-to-tensor ratio,

$$n_{\text{grav}} \equiv \frac{d \ln \mathcal{P}_{\text{grav}}}{d \ln k} = -2\epsilon_v \frac{1 - 2\nu}{(1 - \nu)^2}, \quad (3.33)$$

$$r \equiv \frac{\mathcal{P}_{\text{grav}}}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon_v \frac{1}{(1-\nu)^2}. \quad (3.34)$$

Hence, the consistency relation can be expressed as

$$r = -8n_{\text{grav}} \frac{1}{1-2\nu}. \quad (3.35)$$

Evidently, all quantities will recover the standard forms of slow-roll inflation when  $\nu$  approaches zero. In addition, since  $\nu$  is certainly a small quantity, we thus have  $n_s - 1 \approx -6\epsilon_v + 2\eta_v(1+\nu)$ ,  $\alpha_s \approx 16\epsilon_v\eta_v(1+\nu) - 24\epsilon_v^2(1+\nu) - 2\xi_v^2(1+2\nu)$ ,  $r \approx 16\epsilon_v(1+2\nu)$ , and so on.

For concreteness, we consider a simple inflation model, chaotic inflation, for example. The chaotic inflation has the potential of the form  $V(\phi) \propto \phi^\alpha$  with  $\alpha$  a positive integer. The slow-roll parameters can be easily expressed as  $\epsilon_v = \alpha^2/2\phi^2$  and  $\eta_v = 2(\alpha-1)\epsilon_v/\alpha$ . Strictly speaking, the termination of the inflation should be determined by the condition  $\epsilon_H \simeq 1$ , where  $\epsilon_H \sim \epsilon_v(1-2\nu)/(1-\nu)$ . Nevertheless, the fact that  $\nu$  is a small quantity implies  $\epsilon_N \simeq \epsilon_v$  showing the usual termination condition  $\epsilon_v \simeq 1$  is also appropriate. In any case, we have  $\phi_{\text{end}} \ll \phi_N$ , where  $\phi_N$  (with  $N \sim 60$ ) marks the value of the field when our observable universe leaves the horizon during the inflation. Using Eq. (3.24) and integrating out  $N$ , we derive

$$\phi_N = \sqrt{\frac{2\alpha N}{(1 - \frac{2}{\alpha+2}\nu)}}. \quad (3.36)$$

Consequently, the concrete expressions of the slow-roll parameters can be obtained,

$$\epsilon_v = \frac{\alpha}{4N} \left( 1 - \frac{2}{\alpha+2}\nu \right), \quad (3.37)$$

$$\eta_v = \frac{\alpha-1}{2N} \left( 1 - \frac{2}{\alpha+2}\nu \right), \quad (3.38)$$

$$\xi_v^2 = \frac{(\alpha-1)(\alpha-2)}{4N^2} \left( 1 - \frac{2}{\alpha+2}\nu \right)^2. \quad (3.39)$$

Then we get all quantities of interest in cosmology, such as scalar spectral index and its running as well as the tensor-to-scalar ratio, using Eqs. (3.30), (3.31) and (3.34).

### 3.5 Can LQC effects be observable?

Since inflation manifests fluctuations that were once on scales of quantum gravity dominance to scales of the observable horizon, quantum gravity physics could potentially leave its imprint on the CMB sky. It is of interest to discuss the possibility of detecting the signature of LQC physics in the angular power spectrum. Discussions in the previous subsection have revealed the possible forms of the primordial power spectrum and other observational quantities of interest. The expected amplitude of quantum gravity effects can be characterized by a parameter  $\nu$  which appears in the resulting quantities in cosmology from slow-roll inflation. Naively, one can determine the value of  $\nu$  by fitting the theoretical results to observational data.

Actually, the amplitude of  $\nu$  can easily be constrained using the information of the COBE normalization [28]. The COBE normalization corresponds to  $\delta_H = (2/5)\mathcal{P}_{\mathcal{R}}^{1/2} = 1.91 \times 10^{-5}$  for the mode which crossed the Hubble radius about 60  $e$ -folds before the end of inflation. Comparing this value to Eq. (3.26) gives

$$\frac{V^{1/4}(1-\nu)^{3/4}}{\epsilon_v^{1/4}} = 2.7 \times 10^{-2} M_{\text{Pl}} = 6.7 \times 10^{16} \text{ GeV}, \quad (3.40)$$

where  $M_{\text{Pl}} = 2.4 \times 10^{18} \text{ GeV}$  has been used. The value of the critical density in the effective theory of LQC can be deduced from Eq. (2.8) as  $\rho_{\text{crit}} \approx 0.82\rho_{\text{Pl}}$ . The Planck density  $\rho_{\text{Pl}} = G^{-2} = 2.22 \times 10^{76} \text{ GeV}^4$ , so we have  $\rho_{\text{crit}}^{1/4} = 1.162 \times 10^{19} \text{ GeV}$ . Using Eqs. (3.22) and (3.40), and in view of  $\nu$  is a small quantity, we get

$$\nu \simeq 10^{-9} \epsilon_v. \quad (3.41)$$

Since  $\epsilon_v$  is much less than 1, the LQC parameter  $\nu$  is certainly much smaller than  $10^{-9}$ . Therefore, we conclude that the loop quantum effects can only lead to a very tiny imprint in the primordial power spectrum and their signature can hardly be detected in the CMB sky using present and upcoming observational data.

## 4. Conclusions

In this paper we have investigated the inflationary universe in the framework of the effective theory of LQC. In LQC, the nonperturbative quantum geometry effects lead to a  $\rho^2$  term with a negative sign in the modified Friedmann equation. This semiclassical theory gives rise to a quantum bounce in the high energy regime when the loop quantum effects are dominative power, which is in accordance with the result from the quantum evolution in LQC. The classical big-bang singularity is thus replaced by the quantum bounce. After the bounce, a super-inflation phase was emergent in a natural way. Then the universe underwent a normal inflation stage.

No matter what matter component (even if radiation-like or dust-like matter) dominated the universe in the early era after the bounce, the super-inflation and the subsequent normal inflation would both happen deterministically, since the quantum gravity dynamics led to effective quintom behavior for the universe. The effective equation of state  $w_{\text{eff}}$  crossed the phantom divide (i.e.,  $w_{\text{eff}}$  evolved from  $< -1$  to  $> -1$ ) in the LQG epoch. The super-inflation took place in the range of  $1/2 < x < 1$  and the subsequent normal inflation happened in  $0 < x < 1/2$ . It has been shown that matter with  $w > -1/3$  can proceed to drive the normal inflation for period of time, though it can last at most several  $e$ -folds. This peculiar behavior also manifests the quantum geometry effects in LQC. For  $-1 < w < -1/3$ , the inflation would always take place, thus a reheating mechanism is needed for terminating the inflation.

Since the cosmological perturbations might be generated when the universe was in the quantum gravity regime, the LQG physics could potentially leave its imprint on the CMB sky. We studied the slow-roll inflation using the semiclassical theory of LQC, namely the



modified Friedmann equation inspired from LQC. Taking the LQC effects into account, the power spectra of curvature perturbations and gravitational waves have been derived. We showed that the scalar spectral index, its running and the tensor-to-scalar ratio can be expressed in terms of slow-roll parameters as well as the LQC parameter  $\nu$ . Note that we analyzed the slow-roll inflation only using the modified Friedmann equation derived from LQC, ignoring some LQG corrections in the perturbation equations. More sophisticated analysis on the cosmological perturbation theory with LQG corrections see Refs. [29, 30]. The main effects of LQC have been, however, involved in our results. By analyzing the power spectrum of the curvature perturbations with the information of the COBE normalization, we showed that the imprint of the loop quantum effects is too weak to be observed by present and upcoming observational data.

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